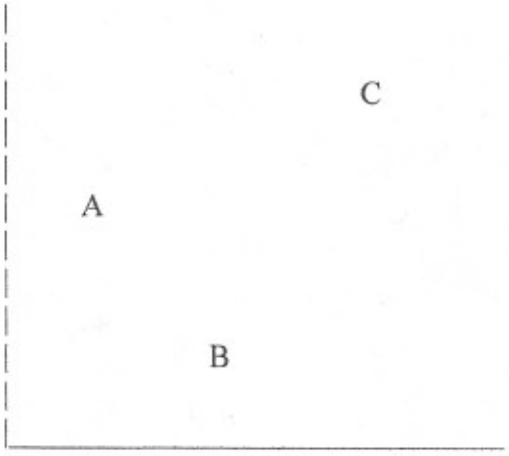
Appendix - Axis Arithmetic

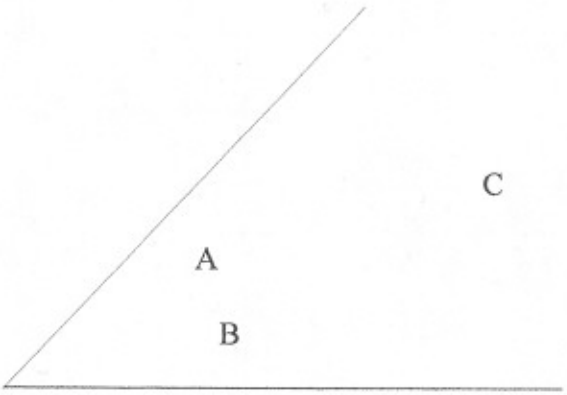
Calculating the distance between two points in a space with hundreds of dimensions, all with varying angles between them, requires some special arithmetic. The convenient solution is to calculate the distances between projections of points' positions onto 2 dimensional planes (all of which are formed by 2 axes related by a known angle), and then use a parametric form of the composite distance formula to combine these distances. In this way the effect of non-orthogonality is limited to the initial calculation between axis pairs.

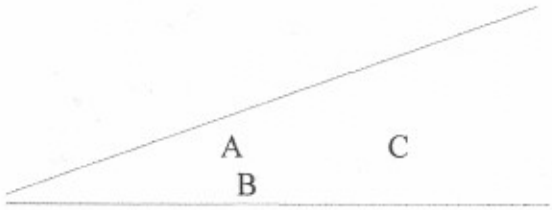


The distance between A and B is found by the usual method, taking the square root of the sum of the squares of the axial distances. Let's call it OD for "original distance":

OD = sqrt((Xa-Xb)^2+(Ya-Yb)^2);

As the angle between these 2 axes decreases, a calculation for AB must produce a smaller and smaller result for the distance, while changing the distance BC or AC.much less.





Clearly the effect of the rotation of the "Y" axis must be maximized when the slope of the line between the two points is -1, and minimized when it is 1. This is accomplished by multiplying OD by a function both of the angle between the axes (angle1) and of the angular relation of the two points:

Adjusted distance = OD *(1 - (cos(angle1) / slopeFunc))

where slopeFunc = 1 / (1 - cos(angle2)) and where angle2 is that which relates the line defined by the points, and the line with a slope=1.

The distance in multiple dimensions is then given by combining those on the several axis pairs. If MDD is this multi-dimensional distance and Dab, Dac...are the distances calculated for projections of the points onto the planes ab, ac,...., and if the number of dimensions is Nd, then

MDD = sqrt((Dab^2 + Dac^2 + . . .) / (Nd -1))

Straightforward algebraic reduction shows that this expression is equivalent to the multi-dimensional distance calculation in its Pythagorean form

MDD = sqrt((Xa-Xb)^2+(Ya-Yb)^2+.....)