Homework 4 **Name:**
Due date: Wednesday, 1/30/08
Weight: 10 points. **Grade:**

There are two common forms of bad proof you need to learn to avoid. The first is proof by example:

Theorem 1. *For real numbers* x, $x^2 - 1 = (x-1)(x+1)$.

INVALID proof (by example). For $x = 4$, $x^2 - 1 = 16 - 1 = 15$, and $(x-1)(x+1) = 3 \cdot 5 = 15$, so the two are equal. For $x = 10$, $x^2 - 1 = 100 - 1 = 99$, and $(x-1)(x+1) = 9 \cdot 11 = 99$, so the two are again equal. The formula holds. □

No proof of this style is ever correct, because no reasoning about specific cases can ever address all possible numbers. If you submit such a thing on homework, it will be returned with nasty comments.

The second flavor of wrong proof is "proof by assumption." This technique involves assuming the thing desired, doing a few calculations, and declaring victory. Of course, it is never acceptable to prove a theorem by simply assuming it!

Theorem 2. *There is a number* x *which is a solution to the equation* $x^2 - 5x + 6 = 0$.

INVALID proof (by assumption). Let r be a solution to the equation. Notice that $r^2 - 5r + 6 = 0$. Thus the equation has a solution. □

Notice that the line "Let r be a solution to the equation" implicitly assumes there is a solution, which is precisely what we're trying to prove.

The next theorem and its proof have similarities to both of the erroneous proofs above. You may find it quite a challenge to read properly:

Theorem 3. *If* n *is a whole number, then* $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$.

Proof. We wish to show that the equation is true for all n. First we test the case $n = 1$. The left hand side is simply 1, and the right hand side is $\frac{1(1+1)}{2} = 1$. Therefore they agree, and the equation is true for $n = 1$.

Assume the equation is true for n. That is, assume $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$. We will show that it is also true for $n+1$. That is, we will show $1 + 2 + \ldots + n + (n+1) = \frac{(n+1)((n+1)+1)}{2}$. We use a chain of equalities:

$1 + 2 + \ldots + n + (n+1) =$

$\frac{n(n+1)}{2} + (n+1) =$

$= \frac{(n+1)((n+1)+1)}{2}$.

This demonstrates that the assertion is true not only for n, but also for $n+1$. Therefore it is true for all values of n. □

(1) Complete the chain of equalities, using algebraic manipulations. If you run out of vertical space, start another column.

(2) What justifies the equality between the first and second lines of the chain of equalities?

(3) Find a step in the proof that gives the appearance of "proof by example." Circle it and mark it with a star ⋆.

(4) Identify a step in the proof that gives the appearance of "proof by assumption." Mark it with a "dagger" †.

(5) Give your honest opinion of this proof.