

**Homework 1****Due date:** Wednesday, 1/23/08**Weight:** 10 points.**Name:****Grade:**

Read this proof with extreme care. Answer all the questions about it, using this sheet and the back. Answer carefully and neatly. I encourage you to type your answers, but it is not required.

**Theorem 1.** *There are infinitely many prime numbers.*

*Proof.* Suppose there were only finitely many primes. Label them  $p_1, p_2, \dots, p_k$ . Let  $n = p_1 \cdot p_2 \cdot \dots \cdot p_k + 1$ . Because  $n$  is one more than a multiple of each  $p_i$ , it is not divisible by any  $p_i$ . Thus  $n$  is not divisible by any prime. But that's impossible – every number is either prime or a product of primes. Therefore there are infinitely many primes.  $\square$

(Another version of this proof, from a text called “Proofs from the book,” is available on google books.) Now reread the proof. Reread it again. It should be essentially memorized.

- (1) Is the theorem statement true? What other knowledge do you have that relates to the theorem statement? Does your prior experience support the theorem statement?
- (2) There is no need to worry further about a theorem if it is false. Can you prove it wrong?
- (3) Is the proof correct? If it has any flaws, point them out.
- (4) Use a google search to find the largest known prime number. What is it? Is the fact that there is a largest prime compatible with the theorem? What gives?
- (5) Make a list of prior knowledge which must be shared between the proof author and the proof reader. Make sure each kind of knowledge is expressed as a fact, not as a procedure, process, or method.