

- (1) Sets (chap 2)
 - (a) Notation
 - (i) listed set notation $\{1, 2, 3, 4\}$, $\{2, 4, 6, 8, \dots\}$
 - (ii) standard names (\mathbb{R} , \mathbb{Q} , \mathbb{C} , \mathbb{Z} , \mathbb{N} , ω , \emptyset)
 - (iii) The magic word is “Let.”
 - (iv) intersect, union, complement, setminus
 - (v) cardinality
 - (vi) set builder – comprehension
 - (vii) set builder – replacement
 - (viii) \in and \subseteq .
 - (b) Direct proof by element chase.
 - (c) Refutation by counterexample
 - (d) Proof by contradiction. (aesthetically worse.)
- (2) Logic (chap 1)
 - (a) The need for systematic logic.
 - (i) to cast away doubt
 - (ii) to make difficult linguistic calculations mathematical instead.
 - (b) Topics of logic: and, or, not, implies, forall, exists.
 - (c) Notations for logical operators.
 - (d) Truth tables, including \rightarrow
 - (e) Tautologies
- (3) Sets 2 (chap 5)
 - (a) Power set
- (4) Cartesian products and relations (chap 3)
 - (a) Cartesian products of two or more sets
 - (b) cardinalities of cartesian products.
 - (c) comparison to vectors.
 - (d) relations
 - (i) graphs of relations
 - (ii) reflexivity, symmetry, transitivity
 - (iii) equivalence relations
 - (iv) modding out by equivalence relations
 - (v) well definedness.
 - (vi) partial orders.
 - (vii) total orders.
- (5) Functions (chap 4)
 - (a) Syntax for evaluation: $f(x)$
 - (b) Syntax for definition: Let $f : D \rightarrow R$ via $f(x) = \dots$
 - (c) Domain
 - (d) Range
 - (e) Injectivity
 - (f) Surjectivity
 - (g) Bijectivity
 - (h) Composition
 - (i) Inverse
 - (j) preimage and image.
- (6) Infinity and induction
 - (a) Induction
 - (i) Fundamental principle of induction
 - (ii) base case.
 - (iii) Two expressions:
 - (A) For contradiction, take least n with $\neg P(n + 1)$
 - (B) $P(n) \rightarrow P(n + 1)$ with base case.
 - (iv) Examples
 - (A) Algebraic examples
 - (B) $2^n \times 2^n$ checkerboard tiling (Golomb’s proof. There’s an applet.)
 - (C) Leaves in a bag.
 - (D) Interesting numbers
 - (E) $8^n 3^n$ is divisible by 5. But factor!
 - (b) Infinite sets, characterization via injections
 - (c) Do infinite sets really exist?
 - (d) Cardinality of infinite sets
 - (e) Cantor’s diagonalization.