3.5 Derivatives of Trig Functions

Answer all questions asked. You may work in pairs. This project is due on Thursday of next week. You may put your answers directly on this handout. Be sure to put your name(s) at the top, and be sure to check out the practice problems listed at the end of the project (we'll discuss them on the Wednesday after break.)

Let's start by trying to find the derivative of $f(x) = \cos x$. Applying the definition of derivative, we get:

$$\frac{d}{dx}\cos x = \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$$

How are we going to simplify this? Seems impossible, but there is a way out. Look up formula 12b in Stewart, appendix D. Use this formula to simplify $\cos(x+h)$. In Stewart's formula, replace y with h.

1. What do you get $\cos(x+h) = ?$

2. Use your answer to the previous question to rewrite $\frac{d}{dx}\cos x = \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$ as $\lim_{h\to 0}\left(\cos(x)\frac{\cos(h)-1}{h} - \sin(x)\frac{sin(h)}{h}\right)$.

   Show all algebra steps used to make this simplification.

3. Now, we can use the difference rule of limits (Limit Law number 2, page 84), and the fact that neither $\sin x$ nor $\cos x$ depend on h to rewrite $\frac{d}{dx}\cos x$ as follows:

$$\frac{d}{dx}\cos x = (\cos x)\left(\lim_{h\to 0}\frac{\cos(h)-1}{h}\right) - (\sin x)\left(\lim_{h\to 0}\frac{\sin(h)}{h}\right)$$

So, finding the derivative of $f(x) = \cos(x)$ boils down to computing the two limits: $\lim_{h\to 0}\frac{\sin(h)}{h}$ and $\lim_{h\to 0}\frac{\cos(h)-1}{h}$.

Use tables of appropriate values, and graphs to estimate these limits. Show your tables and graphs below. (These limits are carefully computed in your text on pp. 171-172. You can look through these computations later if you are interested. The Squeeze Theorem is needed.)

4. Fill in the remaining step to show that $\frac{d}{dx}\cos(x) = -\sin(x)$.

Similar reasoning can be used to show that $\frac{d}{dx}\sin(x) = \cos(x)$. Your text goes through the reasoning on pp. 171-172. (Remember: DESIgN positive: the derivative of *sin* is positive 1 times $\cos(x)$.)

5. *The derivative of tan(x).*

   Use the fact that $\tan x = \frac{\sin x}{\cos x}$ and the quotient rule to show that $\frac{d}{dx}(\tan x) = \sec^2 x$. At some point in your computation, you will have to use the identity $\cos^2 x + \sin^2 x = 1$. Remember too that $1/\cos x = \sec x$.